jets. Two other cases are provided for comparison. One case is the multijet array with suppressed screech and the other case is a single equivalent rectangular jet with screech. The synchronized screech multijet case produced the maximum mass-flux augmentation followed by the screech suppressed multijet and the screeching single equivalent jet. Future work should focus on investigating synchronized screech excitation for smaller internozzle spacings that are useful for mixer-ejector applications.

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References

[1]Glass, D. R., "Effects of Acoustic Feedback on the Spread and Decay of Supersonic Jets," *AIAA Journal*, Vol. 6, No. 10, 1968, pp. 1890–1897.

[2]Krothapalli, A., Hsia, Y., Baganoff, D., and Karamcheti, K., "The Role of Screech Tones in Mixing of an Underexpanded Rectangular Jet," *Journal of Sound and Vibration*, Vol. 106, No. 1, 1986, pp. 119–143.

[3]Rice, E. J., and Raman, G., "Enhanced Mixing of a Rectangular Supersonic Jet by Natural and Induced Screech," AIAA Paper 93-3263, 1993.

[4]Raman, G., and Rice, E. J., "Mixing and Noise Benefit Versus Thrust Penalty in Supersonic Jets Using Impingement Tones," AIAA Paper 94-2955, 1994.

[5]Rice, E. J., "Jet Mixer Noise Suppressor Using Acoustic Feedback," NASA-Lewis Research Center, Patents Pending, Application 08/046, 256, filed April 14, 1993 and 08/194, 654, filed Feb. 10, 1994.

[6]Krothapalli, A., Karamcheti, K., Hsia, Y., and Baganoff, D., "Edge Tones in High-Speed Flows and their Applications to Multiple-Jet Mixing," *AIAA Journal*, Vol. 21, No. 7, 1983, pp. 937,938.

[7]Chandrasekhara, M. S., Krothapalli, A., and Baganoff, D., "Mixing Characteristics of an Underexpanded Multiple Jet Ejector," Stanford Univ., JIAA TR-55, Stanford, CA, 1984.

[8]Nagel, R. T., Denham, H. W., and Papathanasiou, A. G., "Supersonic Jet Screech Tone Cancellation," *AIAA Journal*, Vol. 21, No. 11, 1983, pp. 1541–1545.

Effect of Non-Poisson Samples on Turbulence Spectra from Laser Velocimetry

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Introduction

THE estimation of turbulence spectra from "individual realization" laser velocimetry (LV) data is very important in determining the frequency characteristics of turbulent flows and the associated time and length scales of turbulence structure. The turbulence scale information obtained from the spectral estimates can aid in evaluating theoretical and numerical turbulence models. A thorough knowledge of the factors that affect the accuracy of the spectral estimates is essential.

LV data constitute a set of randomly sampled velocity-time data obtained from the light scattered by seed particles passing through the measurement volume. Spectral analysis of such data has been investigated for several years and special techniques are available to obtain the spectral estimates.[1,2] Shapiro and Silverman[3] showed theoretically that alias-free spectral estimates can be obtained if the sampling is Poisson distributed. In applying these techniques to laser velocimetry, it is generally assumed that the particle arrival at the measurement volume is Poisson distributed. Well-controlled LV experiments do exhibit Poisson distributed samples. There are, however, many flow situations where LV data do not show these characteristics, and they deviate from the true Poisson distribution (see Ref. 4). One such "non-Poisson" distribution of samples was encountered by the authors during the axial flow velocity measurements near the reattachment region (about 9-step heights downstream of the step) of a backward-facing step flow facility and is shown in Fig. 1. (A detailed description of the experimental facility, including the LV system, is given in Ref. 4.) Figure 1 shows the probability density function $p(\Delta t)$ of an occurrence of a given interarrival time Δt, displayed on a semilog plot. The variability at large Δt is because of the low number of samples occurring in that range and is typical of an LV experimental observation. The dotted straight line represents the theoretical (Poisson) distribution[4]

$$p(\Delta t) = \nu \exp(-\nu \Delta t)$$

where ν is the mean data rate. A non-Poisson process deviates from this straight line. If the sampling is truly Poisson distributed, then the two curves should overlap.

The problem of non-Poisson sampling is attributed to the complex dynamics of the flow that cause nonhomogeneous distribution of particles in the flow. Sometimes, in the desire to obtain high data rates, one may intentionally try to force a high particle flow rate through the sample volume. This can also cause a nonhomogeneous distribution of seeding and can lead to non-Poisson arrival of data. The effect of non-Poisson sampling on the accuracy of spectral estimates has not been addressed in the literature. This Note presents the results of an investigation into this effect. The study is based on a simulated first-order spectrum, which typifies a one-dimensional turbulence spectrum, and some LV experimental data.

Simulated Data

Generation of both Poisson and non-Poisson distributed samples for a first-order spectrum of the form

$$S(f) = \frac{2}{1 + 10^{-4}(2\pi f)^2}$$

where $S(f)$ is the power spectral density at frequency f, is described in Ref. 4. Non-Poisson distributed Δt were generated at an arbitrary mean data rate of 200 samples/s, by choosing[4]

$$\Delta t_i = [-10 \ln(1 - r_i)/\nu]^2 \qquad i = 1, 2, \ldots$$

where r is a random number between 0 and 1. The probability density function of these Δt is shown in Fig. 2. This Δt distribution closely

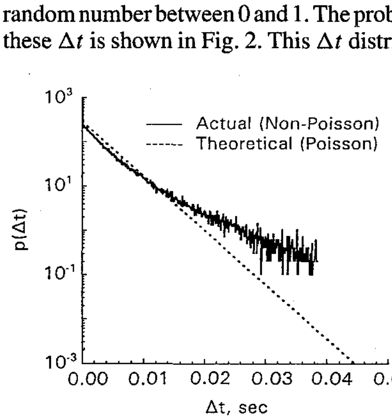

Fig. 1 Probability density function of interarrival times for LV data, ν = 282 samples/s.

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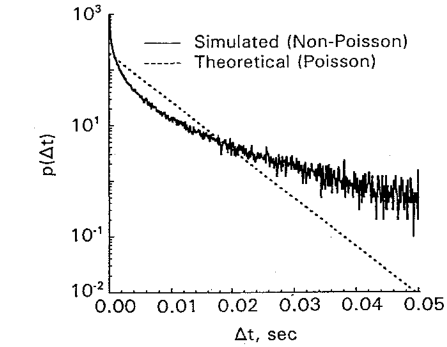

Fig. 2 Probability density function of interarrival times for simulated data, ν = 200 samples/s.

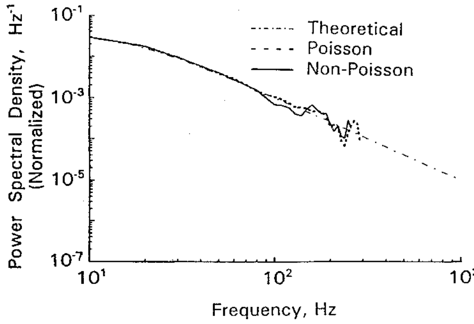

Fig. 3 Power spectra of Poisson and non-Poisson simulated data, ν = 200 samples/s.

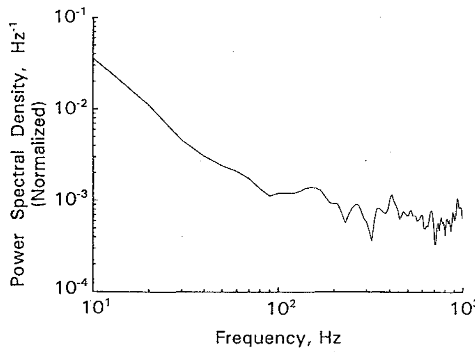

Fig. 4 Power spectrum for LV data referred to in Fig. 1, ν = 282 samples/s.

simulates the non-Poisson samples of the LV data encountered in the present study (see Fig. 1).

Results and Discussion

Power spectral density (PSD) estimates were computed using the slot-correlation technique described in Refs. 1, 2, and 4. Figure 3 shows the PSD estimates (normalized with respect to the signal variance) of the simulated first-order spectrum for both Poisson and non-Poisson sampling cases. The theoretical spectrum is also shown for comparison. The spectral estimates from the Poisson distributed samples follow the theoretical distribution up to about a frequency of 130 Hz. But in the case of non-Poisson distribution, the spectrum starts deviating from the true spectrum at a much lower frequency of about 80 Hz. The spectrum did not improve further even with larger data sets. Also, prefiltering techniques, which are normally applied to improve or extend the frequency range of the spectral estimates, did not show any promise. It has been shown by Sree et al.[4] that, in the case of Poisson sampling, using prefiltering can extend the frequency range up to about three times the mean sampling frequency, or six times the Nyquist frequency, which is amazingly good. However, in the non-Poisson sampling case the frequency range to which reasonable spectral estimates could be made was generally limited to somewhat less than half of the mean sampling frequency, which is just a little less than the Nyquist criterion and, therefore, is not that bad.

Figure 4 shows the normalized spectral estimates of the LV data taken near the shear layer reattachment region in the backward-facing step facility. The spectrum follows the expected shape up to approximately 90 Hz, which is about one-third the mean sampling frequency and two-thirds of the Nyquist criterion. Above that frequency the spectrum appears to be noise. Prefiltering had no effect on the spectral estimates at higher frequencies. The shape of the actual spectrum beyond 90 Hz is quite uncertain in this case. A Poisson process would have revealed the true nature of the spectrum at least up to about the mean sampling frequency and, perhaps, up to about three times that with prefiltering.

Concluding Remarks

Spectral analysis of LV data plays an important role in characterizing a turbulent flow and in estimating the associated turbulence scales, which can be helpful in validating theoretical and numerical turbulence models. The determination of turbulence scales is critically dependent on the accuracy of the spectral estimates. Spectral estimations from "individual realization" LV data are typically based on the assumption of a Poisson sampling process. What this Note has demonstrated is that the sampling distribution must be considered before spectral estimates are used to infer turbulence scales. A non-Poisson sampling process can occur if there is nonhomogeneous distribution of particles in the flow. Based on the study of a simulated first-order spectrum, it has been shown that a non-Poisson sampling process causes the estimated spectrum to deviate significantly from the true spectrum. It is also noted in this case that prefiltering techniques do not improve the spectral estimates at higher frequencies. Further, this Note has addressed only the effect of non-Poisson sampling on the accuracy of the spectral estimates. Effects of other factors such as velocity bias, instrumentation errors, etc. should also be investigated.

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References

[1]Srikantaiah, D. V., and Coleman, H. W., "Turbulence Spectra from Individual Realization Laser Velocimetry Data," *Experiments in Fluids*, Vol. 3, No. 1, 1985, pp. 35–44.

[2]Roberts, J. B., and Ajmani, D. B. S., "Spectral Analysis of Randomly Sampled Signals Using a Correlation-Based Slotting Technique," *Institute of Electrical and Electronic Engineers Proceedings*, Vol. 133, Pt. F, No. 2, 1986, pp. 153–162.

[3]Shapiro, H. S., and Silverman, S. A., "Alias-Free Sampling of Random Noise," *Journal of the Society of Industrial Applied Mathematics*, Vol. 8, No. 2, 1960, pp. 225–248.

[4]Sree, D., Kjelgaard, S. O., and Sellers, W. L., III, "On the Measurement of Turbulence Spectra from non-Poisson Sampled Laser Velocimetry Data," AIAA Paper 94-0041, Jan. 1994.

Static Behavior of Laminated Elastic/ Piezoelectric Plates

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Introduction

EXACT solutions for laminated elastic plates with simple support under transverse load have been developed by Pagano.[1] The resulting displacement and stress distributions demonstrated

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